Spin-charge separation in a nodal antiferromagnetic insulator

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In this Brief Report, by using two-dimensional Hubbard models with π-flux phase and that on a hexagonal lattice as examples, we explore spin-charge-separated solitons in nodal antiferromagnetic (AF) insulator—an AF order with massive Dirac fermionic excitations (see details in text). We calculated fermion zero modes and induced quantum numbers on solitons (half skyrmions) in the continuum limit, which are similar to that in the quasi-one-dimensional conductor polyacetylene $(CH)_x$ and that in topological band insulator. In particular, we find some different phenomena: thanks to an induced staggered spin moment, a mobile half skyrmion becomes a fermionic particle; when a hole or an electron is added, the half skyrmion turns into a bosonic particle with charge degree of freedom only. Our results imply that nontrivial induced quantum number on solitons may be a universal feature of spin-charge separation in different systems.

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I. INTRODUCTION

The Fermi-liquid-based view of the electronic properties has been very successful as a basis for understanding the physics of conventional solids. The quasiparticles of Fermi liquid carry both spin and charge quantum numbers. However, in some cases, spin-charge separation occurs, providing a different framework for thinking about the given systems. It indicates that the systems have two independent elementary excitations, neutral spinon and spinless holon, as opposed to single quasiparticle excitation in conventional solids.

The first example is electronic systems in one spatial dimension.[1] The idea of solitons with induced quantum numbers started with the beautiful result obtained in the context of relativistic quantum field theories by Jackiw and Rebbi.[2] Based on this idea, spin-charge-separated solitons had a lasting impact on condensed-matter physics. In the long molecule chain of trans-polyacetylene, spin-charge separation can occur in terms of soliton states.[3] Due to induced fermion quantum numbers, the soliton may be neutral particles with spin 1/2 or spinless with charge $\pm e$. In two-dimensional (2D) electronic systems, spin-charge separation has been a basic concept in understanding doped Mott-Hubbard insulator related to high-T_c cuprates.[4,5] It is supposed that the particles can be liberated at low energies, with spin-charge separation being an upshot in the "resonating valence bond" (RVB) spin liquid state which was proposed by Anderson[4] as a new state of matter. Recently topological band insulator (TBI) has attracted considerable attention because of its relevance to the quantum spin Hall effect.[6,7] It is pointed out that there exist spin-charge-separated solitons in the presence of π flux with induced quantum numbers.[8–10]

In this Brief Report we focus on a special class of antiferromagnetic (AF) ordered state (nodal AF insulator), and we will show how spin-charge separation occurs. Nodal AF insulator is an AF order (long range or short range) with massive Dirac fermionic excitations. When there is no AF order, fermionic excitations become nodal quasiparticles. There are two examples of nodal AF insulator in condensed-matter physics—one is an AF order on a honeycomb lattice and the other is a π-flux phase together with a nonzero Néel order parameter. Based on these examples, our results confirm that induced quantum number on solitons is an important feature of the spin-charge separation in different systems.

II. FORMULATION

To develop a systematical formulation, we start with the extended Hubbard models,

$$H=-\sum_{\langle i,j\rangle,\sigma} t_{ij}\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{j,\sigma}+U\sum_{j}\hat{n}_{j\uparrow}\hat{n}_{j\downarrow}-\mu\sum_{i,\sigma}\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i,\sigma}+\text{H.c.} \tag{1}$$

Here $\hat{c}^{\dagger}_{i,\sigma}$ and $\hat{c}_{j,\sigma}$ are electronic creation and annihilation operators. U is the on-site Coulomb repulsion. σ are the spin indices for electrons. μ is the chemical potential. $\langle i,j\rangle$ denotes the two sites on a nearest-neighbor link. $\hat{n}_{j\uparrow}$ and $\hat{n}_{j\downarrow}$ are the number operators of electrons with up spin and down spin. On a honeycomb lattice, the nearest-neighbor hopping is a constant, $t_{i,j}=t$; on a square lattice with π-flux phase, it can be chosen as $t_{i,i+\hat{x}}=\chi$, $t_{i,i+\hat{y}}=i\chi$.[11–14] The partition function of the extended Hubbard models is written as $\mathcal{Z}=\int\mathcal{D}\bar{c}\mathcal{D}c\,\exp(-\int_0^{\beta}d\tau L)$, where

$$L=\sum_{j,\sigma}\bar{c}_{j,\sigma}(\partial_{\tau}-\mu)c_{j,\sigma}+\sum_{\langle i,j\rangle,\sigma}t_{ij}\bar{c}_{i,\sigma}c_{j,\sigma}-U\sum_{j}n_{j\uparrow}n_{j\downarrow}. \tag{2}$$

$\bar{c}_{i,\sigma}$ and $c_{j,\sigma}$ are Grassmann variables describing the electronic fields.

First let us derive the long-wavelength effective Lagrangian of the hopping term in the extended Hubbard models. Although π-flux phase does not break translational symmetry, we may still divide the square lattice into two sublattices, A and B. After transforming the hopping term into momentum space, we obtain $E_f=2\chi\sqrt{\cos^2 k_x+\cos^2 k_y}$. So there exist two nodal Fermi points at $\mathbf{k}_1=(\frac{\pi}{2},\frac{\pi}{2})$ and $\mathbf{k}_2=(\frac{\pi}{2},-\frac{\pi}{2})$, and the spectrum of fermions becomes linear in the vicinity of the two nodal points. On a honeycomb lattice, after dividing the lattice into two sublattices, A and B, the dispersion is obtained in Refs. 15–17. There also exist two nodal points,

$\mathbf{k}_1=\frac{2\pi}{\sqrt{3}}(1,\frac{1}{\sqrt{3}})$ and $\mathbf{k}_2=\frac{2\pi}{\sqrt{3}}(-1,-\frac{1}{\sqrt{3}})$, and the spectrum of fermions becomes linear near $\mathbf{k}_{1,2}$. In the continuum limit, the Dirac-type effective Lagrangian describes the low-energy fermionic modes for both cases,

$$\mathcal{L}_f = i\bar{\psi}_1\gamma_\mu\partial_\mu\psi_1 + i\bar{\psi}_2\gamma_\mu\partial_\mu\psi_2, \tag{3}$$

where $\bar{\psi}_1=\psi_1^\dagger\gamma_0=(\bar{\psi}_{\uparrow 1A},\bar{\psi}_{\uparrow 1B},\bar{\psi}_{\downarrow 1A},\bar{\psi}_{\downarrow 1B})$ and $\bar{\psi}_2=\psi_2^\dagger\gamma_0=(\bar{\psi}_{\uparrow 2B},\bar{\psi}_{\uparrow 2A},\bar{\psi}_{\downarrow 2B},\bar{\psi}_{\downarrow 2A})$.[15–17] γ_μ is defined as $\gamma_0=\sigma_0\otimes\tau_z$, $\gamma_1=\sigma_0\otimes\tau_y$, and $\gamma_2=\sigma_0\otimes\tau_x$, where $\sigma_0=\begin{pmatrix}1&0\\0&1\end{pmatrix}$. τ^x, τ^y, and τ^z are Pauli matrices. We have set the Fermi velocity to be of unit $v_F=1$.

In the strong coupling limit, $U\gg t$, there always exists an AF spin-density wave (SDW) order in the extended Hubbard models. Introducing Stratonovich-Hubbard fields for the spin degrees of freedom,[13] we obtain the partition function as $Z=\int\mathcal{D}\bar{c}\mathcal{D}c\mathcal{D}\mathbf{B}\exp(-\int_0^\beta d\tau L)$, where the Lagrangian is given by

$$L=\sum_{j,\sigma}\bar{c}_{j,\sigma}(\partial_\tau-\mu)c_{j,\sigma}+\sum_{\langle i,j\rangle,\sigma}t_{ij}\bar{c}_{i,\sigma}c_{j,\sigma}-\frac{3}{2U}\sum_j\mathbf{B}_j^2 + U\sum_j(-1)^j\mathbf{B}_j\cdot\bar{c}_j\sigma c_j \tag{4}$$

with Pauli matrices $\sigma=(\sigma^x,\sigma^y,\sigma^z)$. Here $\mathbf{B}_j$ is a vector denoting spin configurations, $\mathbf{B}_j=|B_j|\mathbf{n}_j$, where $|B_j|=\phi_0$ represents the value of localized spin moments and $\mathbf{n}_j$ is a unit vector describing the Néel order parameter. In the AF ordered state, the mass gap of the electrons is given as $m=\phi_0 U$. Then starting from Eq. (4), we get the same long-wavelength effective model of nodal AF insulator,[18–20]

$$\mathcal{L}_{\text{eff}}=\sum_\alpha i\bar{\psi}_\alpha\gamma_\mu\partial_\mu\psi_\alpha+m(\bar{\psi}_1\mathbf{n}\cdot\sigma\psi_1-\bar{\psi}_2\mathbf{n}\cdot\sigma\psi_2), \tag{5}$$

where $\alpha=1,2$ labels the two Fermi points.

III. ZERO MODES ON HALF SKYRMIONS

In this section we will study the properties of topological solitons. Instead of considering topological solitons with integer topological charge (skyrmions), we focus on solitons with a half topological charge, $\int \mathbf{d}^2\mathbf{r}\frac{1}{4\pi}\epsilon_{0\nu\lambda}\mathbf{n}\cdot\partial^\nu\mathbf{n}\times\partial^\lambda\mathbf{n}=\frac{1}{2}$. Such soliton is called a half skyrmion (meron). A meron with a narrow core size (the lattice size a) is characterized by $\mathbf{n}=\mathbf{r}/|\mathbf{r}|$, with $\mathbf{r}^2=x^2+y^2\to\infty$.[21–28] In the core of a meron, $|\mathbf{r}|\sim a$, we have $\mathbf{n}\to(0,0,1)$. To stabilize such a soliton, one may add a small easy-plane anisotropy on the Néel order.

Around a meron configuration, the fermionic operators are expanded as

$$\hat{\psi}_\alpha(\mathbf{r},t)=\sum_{\mathbf{k}\neq 0}\hat{b}_{\alpha\mathbf{k}}e^{-iE_\mathbf{k}t}\psi_{\alpha\mathbf{k}}(\mathbf{r})+\sum_{\mathbf{k}\neq 0}\hat{d}^\dagger_{\alpha\mathbf{k}}e^{iE_\mathbf{k}t}\psi^\dagger_{\alpha\mathbf{k}}(\mathbf{r})+\hat{a}^0_\alpha\psi^0_\alpha(\mathbf{r}), \tag{6}$$

where $\hat{b}_{\alpha\mathbf{k}}$ and $\hat{d}^\dagger_{\alpha\mathbf{k}}$ are operators of $\mathbf{k}\neq 0$ modes that are irrelevant to the soliton states discussed below. $\psi^\dagger_{\alpha\mathbf{k}}(\mathbf{r})=(\psi^{0*}_{\uparrow\alpha\mathbf{A}k},\psi^{0*}_{\uparrow\alpha B\mathbf{k}},\psi^{0*}_{\downarrow\alpha\mathbf{A}\mathbf{k}},\psi^{0*}_{\downarrow\alpha B\mathbf{k}})$ are the functions of zero modes. $\hat{a}^0_\alpha$ are annihilation operators of zero modes.

To obtain the zero modes, we write down two Dirac equations from Eq. (5),

$$i\partial_x\gamma_1\psi^0_1+i\partial_y\gamma_2\psi^0_1+m\mathbf{n}\cdot\sigma\psi^0_1=0 \tag{7}$$

and

$$i\partial_x\gamma_1\psi^0_2+i\partial_y\gamma_2\psi^0_2-m\mathbf{n}\cdot\sigma\psi^0_2=0. \tag{8}$$

First we solve the Dirac equation for ψ^0_1. With the ansatz

$$\psi^0_1=\begin{pmatrix}\xi_1(\tilde{x})e^{-i\theta}\\ \xi_2(\tilde{x})\\ \xi_3(\tilde{x})\\ \xi_4(\tilde{x})e^{i\theta}\end{pmatrix},$$

we have

$$\partial_{\tilde{x}}\xi_2=\xi_3,\quad \partial_{\tilde{x}}\xi_3=\xi_2,$$

$$\partial_{\tilde{x}}\xi_1=-\frac{\xi_1}{\tilde{x}}+\xi_4,\quad \partial_{\tilde{x}}\xi_4=-\frac{\xi_4}{\tilde{x}}+\xi_1, \tag{9}$$

where $\mathbf{r}=|\mathbf{r}|(\cos\theta,\sin\theta)$ and $\tilde{x}=\frac{|\mathbf{r}|}{m}$. The solution has been obtained in Ref. 29 as

$$\xi_1(\tilde{x})=\xi_4(\tilde{x})=0,\quad \xi_2(\tilde{x})=\xi_3(\tilde{x})=\exp(-\tilde{x}). \tag{10}$$

So the solution of ψ^0_1 becomes

$$\begin{pmatrix}0\\ \exp(-\tilde{x})\\ \exp(-\tilde{x})\\ 0\end{pmatrix}.$$

To solve ψ^0_2, we transform the equation $i\partial_i\gamma_i\psi^0_2-m\mathbf{n}\cdot\sigma\psi^0_2=0$ into

$$Ui\partial_i\gamma_i\tilde{\psi}^0_2U^{-1}+mU\mathbf{n}\cdot\sigma\tilde{\psi}^0_2U^{-1}=0, \tag{11}$$

where $U=e^{i\pi\gamma_0/2}$, $U\gamma_iU^{-1}=-\gamma_i$, and $U^{-1}\psi^0_2U=\tilde{\psi}^0_2$. Then the solution of ψ^0_2 is obtained as

$$\begin{pmatrix}0\\ -\exp(-\tilde{x})\\ \exp(-\tilde{x})\\ 0\end{pmatrix}.$$

It is noticeable that from the above solutions of zero modes, the components $\psi^0_{\uparrow 1A}$, $\psi^0_{\downarrow 1B}$, $\psi^0_{\downarrow 2A}$, and $\psi^0_{\uparrow 2B}$ are all zero.

IV. TOPOLOGICAL MECHANISM OF SPIN-CHARGE SEPARATION

For the solutions of zero modes, there are four zero-energy soliton states $|\text{sol}\rangle$ around a half skyrmion which are denoted by $|1_+\rangle\otimes|2_+\rangle$, $|1_-\rangle\otimes|2_-\rangle$, $|1_-\rangle\otimes|2_+\rangle$, and $|1_+\rangle\otimes|2_-\rangle$. $|1_-\rangle$ and $|2_-\rangle$ are empty states of the zero modes $\psi^0_1(\mathbf{r})$ and $\psi^0_2(\mathbf{r})$; $|1_+\rangle$ and $|2_+\rangle$ are their occupied states. Thus we have the relationship between $\hat{a}^0_\alpha$ and $|\text{sol}\rangle$ as

$$\hat{a}^0_1|1_+\rangle=|1_-\rangle,\quad \hat{a}^0_1|1_-\rangle=0,\quad \hat{a}^0_2|2_+\rangle=|2_-\rangle,\quad \hat{a}^0_2|2_-\rangle=0. \tag{12}$$

First we define the total induced fermion number operators of the soliton states, $\hat{N}_F=\Sigma_\alpha \hat{N}_{\alpha,F}$, with

$$\hat{N}_{\alpha,F} \equiv \int :\hat{\psi}_\alpha^\dagger \hat{\psi}_\alpha: d^2\mathbf{r} = (\hat{a}_\alpha^0)^\dagger \hat{a}_\alpha^0 + \sum_{\mathbf{k}\neq 0} (\hat{b}_{\alpha\mathbf{k}}^\dagger \hat{b}_{\alpha\mathbf{k}} - \hat{d}_{\alpha\mathbf{k}}^\dagger \hat{d}_{\alpha\mathbf{k}}) - \frac{1}{2}. \tag{13}$$

$::\hat{\psi}_\alpha^\dagger \hat{\psi}_\alpha::$ means normal product of $\hat{\psi}_\alpha^\dagger \hat{\psi}_\alpha$. From the relation between $\hat{a}_\alpha^0$ and $|\mathrm{sol}\rangle$ in Eq. (12), we find that $|1_\pm\rangle$ or $|2_\pm\rangle$ have eigenvalues of $\pm\frac{1}{2}$ of the total induced fermion number operator $\hat{N}_F$,

$$\hat{N}_{1,F}|1_\pm\rangle = \pm\frac{1}{2}|1_\pm\rangle, \quad \hat{N}_{1,F}|2_\pm\rangle = 0,$$

$$\hat{N}_{2,F}|2_\pm\rangle = \pm\frac{1}{2}|2_\pm\rangle, \quad \hat{N}_{2,F}|1_\pm\rangle = 0. \tag{14}$$

Another important induced quantum number operator is the staggered spin operator, $\hat{S}_{(\pi,\pi)}^z=\frac{1}{2}\Sigma_{i\in A}\hat{c}_i^\dagger \sigma_z \hat{c}_i - \frac{1}{2}\Sigma_{i\in B}\hat{c}_i^\dagger \sigma_z \hat{c}_i = \frac{1}{2}\int :[(\hat{\psi}_{\uparrow 1A}^\dagger \hat{\psi}_{\uparrow 1A} + \hat{\psi}_{\downarrow 1B}^\dagger \hat{\psi}_{\downarrow 1B} - \hat{\psi}_{\downarrow 1A}^\dagger \hat{\psi}_{\downarrow 1A} - \hat{\psi}_{\uparrow 1B}^\dagger \hat{\psi}_{\uparrow 1B}) + (\hat{\psi}_{\uparrow 2A}^\dagger \hat{\psi}_{\uparrow 2A} + \hat{\psi}_{\downarrow 2B}^\dagger \hat{\psi}_{\downarrow 2B} - \hat{\psi}_{\downarrow 2A}^\dagger \hat{\psi}_{\downarrow 2A} - \hat{\psi}_{\uparrow 2B}^\dagger \hat{\psi}_{\uparrow 2B})]: d^2\mathbf{r}$. For the four degenerate zero modes, it can be simplified into $\hat{S}_{(\pi,\pi)}^z|\mathrm{sol}\rangle = \frac{1}{2}(\hat{N}_{2,F}-\hat{N}_{1,F})|\mathrm{sol}\rangle$. Let us show the detailed calculations. From the zero solutions of $\psi_{\uparrow 1A}^0$, $\psi_{\downarrow 1B}^0$, $\psi_{\downarrow 2A}^0$, and $\psi_{\uparrow 2B}^0$, we obtain the following four equations:

$$\left(\int :\hat{\psi}_{\uparrow 1A}^\dagger \hat{\psi}_{\uparrow 1A}: d^2\mathbf{r}\right)|\mathrm{sol}\rangle \equiv 0,$$

$$\left(\int :\hat{\psi}_{\downarrow 1B}^\dagger \hat{\psi}_{\downarrow 1B}: d^2\mathbf{r}\right)|\mathrm{sol}\rangle \equiv 0,$$

$$\left(\int :\hat{\psi}_{\downarrow 2A}^\dagger \hat{\psi}_{\downarrow 2A}: d^2\mathbf{r}\right)|\mathrm{sol}\rangle \equiv 0,$$

$$\left(\int :\hat{\psi}_{\uparrow 2B}^\dagger \hat{\psi}_{\uparrow 2B}: d^2\mathbf{r}\right)|\mathrm{sol}\rangle \equiv 0. \tag{15}$$

Using the above four equations, we obtain

$$\begin{aligned}\hat{S}_{(\pi,\pi)}^z|\mathrm{sol}\rangle &= \int d^2\mathbf{r}\Big[-\frac{1}{2}:(\hat{\psi}_{\uparrow 1A}^\dagger \hat{\psi}_{\uparrow 1A} + \hat{\psi}_{\downarrow 1B}^\dagger \hat{\psi}_{\downarrow 1B} + \hat{\psi}_{\downarrow 1A}^\dagger \hat{\psi}_{\downarrow 1A} \\ &\quad + \hat{\psi}_{\uparrow 1B}^\dagger \hat{\psi}_{\uparrow 1B}): + \frac{1}{2}:(\hat{\psi}_{\uparrow 2A}^\dagger \hat{\psi}_{\uparrow 2A} + \hat{\psi}_{\downarrow 2B}^\dagger \hat{\psi}_{\downarrow 2B} \\ &\quad + \hat{\psi}_{\downarrow 2A}^\dagger \hat{\psi}_{\downarrow 2A} + \hat{\psi}_{\uparrow 2B}^\dagger \hat{\psi}_{\uparrow 2B}):|\mathrm{sol}\rangle\Big] \\ &= -\frac{1}{2}(\hat{N}_{1,F}-\hat{N}_{2,F})|\mathrm{sol}\rangle.\end{aligned}$$

Then we calculate the two induced quantum numbers defined above. Without doping, the soliton states of a half skyrmion are denoted by $|1_-\rangle\otimes|2_+\rangle$ and $|1_+\rangle\otimes|2_-\rangle$. One can easily check that the total induced fermion number on the solitons is zero from the cancellation effect between the two nodals $\hat{N}_F|1_-\rangle\otimes|2_+\rangle=\hat{N}_F|1_+\rangle\otimes|2_-\rangle=0$. It is consistent with the earlier results to forbid a Hopf term for the low-energy theory of two-dimensional Heisenberg model.[30] On the other hand, there exists *an induced staggered spin moment* on the soliton states $|1_-\rangle\otimes|2_+\rangle$ and $|1_+\rangle\otimes|2_-\rangle$,

$$\hat{S}_{(\pi,\pi)}^z|1_-\rangle\otimes|2_+\rangle = \frac{1}{2}|1_-\rangle\otimes|2_+\rangle,$$

$$\hat{S}_{(\pi,\pi)}^z|1_+\rangle\otimes|2_-\rangle = -\frac{1}{2}|1_+\rangle\otimes|2_-\rangle. \tag{16}$$

The induced staggered spin moment may be straightforwardly obtained by combining the definition of $\hat{S}_{(\pi,\pi)}^z$ and Eq. (14) together.

When half skyrmions become mobile, their quantum statistics becomes important. The half skyrmions show similar behavior of vortices in the *XY* model; it can move on dual lattices and feel an effective π-flux phase. One can see the detailed kinematics of the topological excitations in Refs. 23 and 25.

Let us examine the statistics of a half skyrmion with an induced staggered spin moment. In CP(1) representation of $\mathbf{n}$, a "bosonic spinon" is introduced by $\mathbf{n}=\bar{\mathbf{z}}\sigma\mathbf{z}$ with $\mathbf{z}=\binom{z_\uparrow}{z_\downarrow}$ and $\bar{\mathbf{z}}\mathbf{z}=1$. Since each bosonic spinon $\mathbf{z}$ carries $\frac{1}{2}$ staggered spin moment, an induced staggered spin moment corresponds to a trapped bosonic spinon $\mathbf{z}$. On the other hand, a half skyrmion can be regarded as a π flux of the bosonic spinon, $\frac{1}{2\pi}\int \epsilon_{\mu\nu}\partial_\mu a_\nu d^2\mathbf{r} = \frac{1}{4\pi}\int \mathbf{d^2r}\,\epsilon_{0\nu\lambda}\mathbf{n}\cdot\partial^\nu\mathbf{n}\times\partial^\lambda\mathbf{n}=\frac{1}{2}$, with $a_\mu \equiv \frac{i}{2}(\bar{\mathbf{z}}\partial_\mu\mathbf{z}-\partial_\mu\bar{\mathbf{z}}\mathbf{z})$. To be more explicit, moving a bosonic spinon z around a half skyrmion generates a Berry phase ϕ to $\mathbf{z}\to\mathbf{z}'=\binom{z_\uparrow e^{i\phi}}{z_\downarrow e^{i\phi}}$ where $\phi=\int\epsilon_{\mu\nu}\partial_\mu a_\nu d^2\mathbf{r}=\pi$. As a result, a bosonic spinon $\mathbf{z}$ and a half skyrmion (meron or antimeron) share mutual semion statistics. Binding the trapped bosonic spinon, a mobile half skyrmion becomes *a fermionic particle*. We may use the operator $\hat{f}_\sigma$ to describe such neutral fermionic particle with half spin. The relation between the zero-energy states and the fermionic states is given as $|1_+\rangle\otimes|2_-\rangle=\hat{f}_\downarrow^\dagger|0\rangle_f$ and $|1_-\rangle\otimes|2_+\rangle=\hat{f}_\uparrow^\dagger|0\rangle_f$ (the state $|0\rangle_f$ is defined through $\hat{f}_\uparrow|0\rangle_f=\hat{f}_\downarrow|0\rangle_f=0$). We call such neutral object (fermion with $\pm\frac{1}{2}$ spin degree freedom) a (fermionic) "spinon."

Now we go away from half filling. It is known that when a hole (electron) is doped, it is equivalent to removing (adding) an electron. Without considering the existence of half skyrmions, the hole (electron) will be doped into the lower (upper) Hubbard band. The existence of zero modes on half skyrmions leads to the appearance of bound levels in the middle of the Mott-Hubbard gap.[22] The hole (electron) will be doped onto the bound states on the half skyrmion and then one of the zero modes will be occupied. When one hole is doped, the soliton state is denoted by $|1_-\rangle\otimes|2_-\rangle$. One can easily check the result by calculating its induced quantum numbers (Table I). On one hand, there is no induced staggered spin moment, $\hat{S}_{(\pi,\pi)}^z|1_-\rangle\otimes|2_-\rangle=0$. On the other hand, the total fermion number is not zero, $\hat{N}_F|1_-\rangle\otimes|2_-\rangle=-|1_-\rangle\otimes|2_-\rangle$. These results mean that such soliton state is a spinless "holon" with positive charge degrees of freedom. After bind-

TABLE I. Quantum numbers of the degenerate soliton states.

	$\|1_+\rangle\otimes\|2_+\rangle$	$\|1_-\rangle\otimes\|2_+\rangle$	$\|1_+\rangle\otimes\|2_-\rangle$	$\|1_-\rangle\otimes\|2_-\rangle$
N_F	1	0	0	−1
$S^z_{(\pi,\pi)}$	0	1/2	−1/2	0

ing a fermionic hole, the soliton state (holon) does not have an induced staggered spin moment. Thus the holon obeys bosonic statistics and becomes *charged bosonic particles*. When one electron is doped, the soliton state is denoted by $|1_+\rangle\otimes|2_+\rangle$. The induced quantum numbers of it are $\hat{N}_F|1_+\rangle\otimes|2_+\rangle=+|1_+\rangle\otimes|2_+\rangle$ and $\hat{S}^z_{(\pi,\pi)}|1_+\rangle\otimes|2_+\rangle=0$. Such soliton state is also a bosonic particle with a negative charge but without spin degrees of freedom. We call such a soliton state an "electon" to mark the difference with the word "electron."

Finally we get a *topological mechanism of spin-charge separation* in nodal AF insulators. There exist two types of topological objects—one is the fermionic spinon and the other is the bosonic holon (or the bosonic electon). The existence of a *large* Mott-Hubbard gap of electrons is very important to protect the spin-charge separation in the so-called nodal AF insulator. Near the quantum critical point where the AF order vanishes, $\phi_0\to 0$, the zero mode may disappear and our results cannot be reliable.

In a one-dimensional (1D) system, real spin-charge separation may occur. As far as the low-energy physics is concerned, the spin and charge dynamics are completely decoupled from each other. In 2D, real spin-charge separation in nodal AF insulators cannot occur in the long-range AF order. At high temperature, due to the screening effect, free half skyrmions may exist. In the future we will study the deconfinement condition of spin-charge-separated solitons and explore the properties of deconfined phases with real spin-charge separation.

V. SUMMARY

By using 2D π-flux phase Hubbard model and the Hubbard model on a honeycomb lattice as examples, we explore spin-charge separation in nodal AF insulator. The crux of the matter in this Brief Report is the discovery of induced staggered spin moment $S^z_{(\pi,\pi)}$ on half skyrmions in nodal AF insulators. Based on such nontrivial induced quantum number, we classify four degenerate soliton states with zero energy—two of them ($|1_-\rangle\otimes|2_+\rangle$ and $|1_+\rangle\otimes|2_-\rangle$) represent the up-spin and down-spin states for a fermionic spinon, another state ($|1_-\rangle\otimes|2_-\rangle$) represents a holon, and the last one ($|1_+\rangle\otimes|2_+\rangle$) denotes an electon.

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